Impact of Fermion Mass Degeneracy on Flavor Mixing

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Abstract

We carry out a systematic analysis of flavor mixing and CP violation in the conceptually interesting limit where two quarks or leptons of the same charge are degenerate in mass. We pay some particular attention to the impact of neutrino mass degeneracy and Majorana phase degeneracy on the lepton flavor mixing matrix.

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I. INTRODUCTION

The phenomenon of quark flavor mixing, described by the 3×3 Cabibbo-Kobayashi-Maskawa (CKM) matrix V_{CKM} [1], is attributed to the mismatch between the diagonalization of the up-type quark mass matrix M_{u} and that of the down-type quark mass matrix M_{d}. In the standard model, the weak charged current of quarks can be written as

$$J^{\mu}_{\mathrm{quark}} = \overline{(u,c,t)_{\mathrm{L}}}\, \gamma^{\mu} V_{\mathrm{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{\mathrm{L}}, \tag{1}$$

where (u,c,t) and (d,s,b) denote the mass eigenstates of up-type and down-type quarks, respectively. The phases of six left-handed quark fields in J^{μ}_{quark} can arbitrarily be redefined, but the mass terms

$$\overline{(u,c,t)_{\mathrm{L}}} \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_{\mathrm{R}} \tag{2}$$

and

$$\overline{(d,s,b)_{\mathrm{L}}} \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{\mathrm{R}} \tag{3}$$

do not change if six right-handed quark fields are accordingly rephased. This freedom, together with the requirement that V_{CKM} must be unitary, allows us to parametrize V in terms of four independent parameters, which are commonly taken as three rotation angles and one CP-violating phase [2]. All of these four parameters have been measured in a number of delicate experiments on quark mixing and CP violation [3].

Thanks to the Super-Kamiokande [4], SNO [5], KamLAND [6] and K2K [7] neutrino oscillation experiments, we are now convinced that neutrinos are also massive and lepton flavors are mixed. The lepton flavor mixing matrix V_{MNS}, which is referred to as the Maki-Nakagawa-Sakata (MNS) matrix [8] in some literature, arises from the mismatch between the diagonalization of the charged lepton mass matrix M_l and that of the (effective) neutrino mass matrix M_ν at low energies. Similar to J^{μ}_{quark}, the weak charged current of leptons reads

$$J^{\mu}_{\mathrm{lepton}} = \overline{(e,\mu,\tau)_{\mathrm{L}}}\, \gamma^{\mu} V_{\mathrm{MNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\mathrm{L}}, \tag{4}$$

where (e,μ,τ) and (ν_1,ν_2,ν_3) denote the mass eigenstates of charged leptons and active neutrinos, respectively. If neutrinos are Dirac particles, the MNS matrix V_{MNS} is just a leptonic analogue of the CKM matrix V_{CKM} and can also be parametrized in terms of three mixing angles and one CP-violating phase. Only the two mixing angles relevant to solar and atmospheric neutrino oscillations have so far been determined from a global analysis of current experimental data [9]. If neutrinos are Majorana particles, however, there is no

freedom to redefine the *relative* phases of three left-handed neutrino fields in J^μ_{lepton}. The reason is simply that the effective Majorana mass term

$$\overline{(\nu_1, \nu_2, \nu_3)_{\text{L}}} \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \begin{pmatrix} \nu^{\text{c}}_1 \\ \nu^{\text{c}}_2 \\ \nu^{\text{c}}_3 \end{pmatrix}_{\text{R}} \tag{5}$$

depends nontrivially on the rephasing of left-handed neutrino fields (where $\nu^{\text{c}}_i \equiv C \overline{\nu}^T_i$ with C being the charge-conjugation operator). In this case, two additional CP-violating phases are needed to fully parametrize V_{MNS} – namely, a complete parametrization of V_{MNS} requires three rotation angles and three CP-violating phases [10]. Because neutrinos are expected to be of the Majorana type in most of the promising neutrino mass models (such as those incorporating the seesaw mechanism [11]), we shall focus our attention on the phenomenon of lepton flavor mixing associated with three light Majorana neutrinos in the following.

The parametrization of V_{CKM} or V_{MNS} can concretely be realized with the help of three orthogonal matrices O_{12}, O_{23} and O_{13}, which correspond to simple rotations in the (1,2), (2,3) and (1,3) planes:

$$O_{12}(\theta_{12}) = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} ,$$

$$O_{23}(\theta_{23}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} ,$$

$$O_{13}(\theta_{13}) = \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} , \tag{6}$$

where $s_{ij} \equiv \sin \theta_{ij}$ and $c_{ij} \equiv \cos \theta_{ij}$. Of course, at least one complex phase should properly be included into O_{ij} (e.g., by replacing "1" in the (3,3) position of O_{12} with $e^{i\delta}$ [12]), such that V_{CKM} consists of one nontrivial CP-violating phase and V_{MNS} contains three nontrivial CP-violating phases. For the sake of convenience in subsequent discussions, we list twelve different representations of V_{CKM} and V_{MNS} in Table I.

As pointed out by Jarlskog [13], it is always possible to make one matrix element of V_{CKM} vanishing in the limit where two quarks of the same charge are degenerate in mass. In such an unrealistic but conceptually interesting case, the nontrivial CP-violating phase is also removable from V_{CKM}. One may further speculate (a) how the quark mass degeneracy explicitly affects V_{CKM}; (b) whether similar results can be obtained for V_{MNS} in the limit where two charged leptons or two neutrinos are degenerate in mass; and (c) what impact the degeneracy of two Majorana CP-violating phases together with the degeneracy of two neutrino masses may have on V_{MNS}. Such speculations call for a careful analysis of the underlying correlation between fermion mass degeneracy and flavor mixing.

The purpose of this paper is to present a systematic analysis of the impact of fermion mass degeneracy on flavor mixing and CP violation, in order to answer the above questions. Section II is devoted to quark flavor mixing in the limit of quark mass degeneracy. Detailed discussions about lepton flavor mixing in the limit of lepton mass degeneracy are given in Section III. We draw the conclusion in Section IV.

II. QUARK FLAVOR MIXING

We first discuss how the CKM matrix V_{CKM} can be simplified in the limit where two quarks of the same charge are degenerate in mass. To do so, it is convenient to adopt one of the parametrizations of V_{CKM} listed in Table I. Let us assume u and c quarks to be degenerate as an example. In this case, it would be impossible to distinguish between u and c – in other words, there would be an extra symmetry which allows an arbitrary *unitary* rotation in the space spanned by u and c quarks but keeps the mass term in Eq. (2) unchanged [14]. Making the transformation

$$\begin{pmatrix} u \\ c \\ t \end{pmatrix} \implies \begin{pmatrix} u \\ c \\ t \end{pmatrix}' = O_{12}^{\dagger} \begin{pmatrix} u \\ c \\ t \end{pmatrix} , \tag{7}$$

one may easily verify that the primed fields remain the mass eigenstates of three up-type quarks. In the new basis, the CKM matrix becomes $V'_{\mathrm{CKM}} = O_{12}^{\dagger} V_{\mathrm{CKM}}$. Once parametrizations (A), (B), (G) and (H) in Table I are taken into account, we obtain the corresponding patterns of V'_{CKM} as follows:

$$V'_{\mathrm{CKM}} = \begin{cases} O_{23} \otimes P_1 \otimes \tilde{O}_{12} & \text{(A)} , \\ O_{13} \otimes P_2 \otimes \tilde{O}_{12} & \text{(B)} , \\ O_{23} \otimes P_1 \otimes O_{13} & \text{(G)} , \\ O_{13} \otimes P_2 \otimes O_{23} & \text{(H)} , \end{cases} \tag{8}$$

in which $(V'_{\mathrm{CKM}})_{13} = 0$, $(V'_{\mathrm{CKM}})_{23} = 0$, $(V'_{\mathrm{CKM}})_{12} = 0$ and $(V'_{\mathrm{CKM}})_{21} = 0$, respectively. This result indicates that one of the four off-diagonal matrix elements in the first two rows of V_{CKM} (or equivalently, one of the three mixing angles of V_{CKM}) is removable in the $m_u = m_c$ limit. Because O_{23} (or O_{13}) and P_1 (or P_2) are commutable, the latter can then be rotated away from V'_{CKM} by rephasing the up-type quark mass eigenstates. It becomes obvious that no CP violation can really manifest itself in the $m_u = m_c$ limit. We conclude that there would exist only two flavor mixing angles and the CP symmetry would be conserving, if u and c quarks were degenerate in mass.

The above arguments can straightforwardly be repeated for any pair of up-type or down-type quarks. To be concise, we summarize the relevant results in Table II, where the removable complex phase in V'_{CKM} has been omitted. We see that it is always possible to make one of the six off-diagonal matrix elements of V_{CKM} vanishing in the limit where two quarks of the same charge are degenerate in mass. CP would be a good symmetry in this case.

If both $m_u = m_c$ and $m_d = m_s$ held, one could easily show that the transformation

$$\begin{pmatrix} d \\ s \\ b \end{pmatrix} \implies \begin{pmatrix} d \\ s \\ b \end{pmatrix}' = \tilde{O}_{12} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{9}$$

together with that in Eq. (7) does not change any physics. In the new (primed) basis, the CKM matrix V_{CKM} turns out to be $V''_{\mathrm{CKM}} = O_{12}^{\dagger} V_{\mathrm{CKM}} \tilde{O}_{12}^{\dagger}$. It is then possible to obtain

$$V''_{\mathrm{CKM}} = \begin{cases} O_{23} \otimes P_1 & \text{(A)} , \\ O_{13} \otimes P_2 & \text{(B)} , \end{cases} \tag{10}$$

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for parametrizations (A) and (B) of V_{CKM}. The phase matrix P_1 or P_2 on the right-hand side of V''_{CKM} can easily be rotated away by redefining the relevant phases of down-type quark fields. Eq. (10) indicates that only one of the three mixing angles of V_{CKM} can survive in the limit of $m_u = m_c$ and $m_d = m_s$. The similar argument is applicable for any pair of up-type quarks and any pair of down-type quarks. There are totally nine such possibilities, as summarized in Table III, where the removable complex phase in V''_{CKM} has been omitted.

If three up-type (or down-type) quarks were all degenerate in mass, one would have sufficient freedom to redefine the quark mass eigenstates and transform the CKM matrix V_{CKM} into the unity matrix. Then there would be no quark flavor mixing and CP violation. We conclude that a necessary condition for three-flavor mixing and CP violation in the quark sector is that two quarks of the same charge must not be degenerate in mass. This condition is certainly satisfied in reality, because three quarks in each sector have a strong mass hierarchy [3].

III. LEPTON FLAVOR MIXING

The Majorana nature of (ν_1, ν_2, ν_3) neutrinos makes the description of lepton flavor mixing somehow more complicated than the description of quark flavor mixing. To make our discussions more convenient, we decompose V_{MNS} into a product of U_{MNS} and P_ϕ; i.e., $V_{\text{MNS}} = U_{\text{MNS}} P_\phi$, where U_{MNS} is just a leptonic analogue of the CKM matirx V_{CKM} and $P_\phi = \text{Diag}\{e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}\}$ represents the diagonal Majorana phase matrix containing two nontrivial CP-violating phases [10]. Subsequently we take two steps to discuss the impact of lepton mass degeneracy on lepton flavor mixing.

First, we consider the partial or total mass degeneracy of three charged leptons, whose left-handed and right-handed fields may have completely independent phases. If $m_e = m_\mu$ held, for example, one would be able to make one of the four off-diagonal matrix elements in the first two rows of U_{MNS} vanishing. This situation is exactly the same as the situation of quark mixing in the $m_u = m_c$ limit. Thus the relevant discussions in Section II are applicable for U_{MNS}, as shown in Table IV, where the reduced form of U_{MNS} has been denoted by U'_{MNS}. Note, however, that the Majorana phase matrix P_ϕ is not influenced by the mass degeneracy of charged leptons.

Second, we consider the case in which three neutrinos are partially or totally degenerate in mass. If $m_1 = m_2$ held, for instance, there would be an extra symmetry which allows an arbitrary *orthogonal* rotation in the space spanned by ν_1 and ν_2 neutrinos but keeps the mass term in Eq. (5) unchanged. Namely, the transformation

$$\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \implies \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}' = \tilde{O}_{12} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \tag{11}$$

does not change any physics. In the new basis, the MNS matrix becomes $V'_{\text{MNS}} = V_{\text{MNS}} \tilde{O}_{12}^T = U_{\text{MNS}} P_\phi \tilde{O}_{12}^T$. The explicit parametrizations of V_{MNS} have been listed in Table I. Note that $\tilde{O}_{12}^T$ cannot commute with P_ϕ, unless $\phi_1 = \phi_2$ is taken. Hence V'_{MNS} is *in general* unable to be simplified by the orthogonal transformation in Eq. (11). Such a conclusion can also be drawn for $m_1 = m_3$ and $m_2 = m_3$ cases. It remains valid even in the $m_1 = m_2 = m_3$ case.

We proceed with the decomposition $V_{\mathrm{MNS}} = U_{\mathrm{MNS}}P_\phi$. The Majorana phase matrix P_ϕ can be absorbed by a redefinition of the neutrino mass eigenstates,

$$\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \Longrightarrow \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}' = P_\phi \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} . \tag{12}$$

In the new basis, the neutrino mass term in Eq. (5) turns out to be

$$\overline{(\nu_1, \nu_2, \nu_3)'_{\mathrm{L}}} \begin{pmatrix} m'_1 & 0 & 0 \\ 0 & m'_2 & 0 \\ 0 & 0 & m'_3 \end{pmatrix} \begin{pmatrix} \nu^{\mathrm{c}}_1 \\ \nu^{\mathrm{c}}_2 \\ \nu^{\mathrm{c}}_3 \end{pmatrix}'_{\mathrm{R}} , \tag{13}$$

where $m'_a \equiv m_a e^{2i\phi_a}$ (for $a = 1, 2, 3$). The complex neutrino masses (m'_1, m'_2, m'_3) contain the information about Majorana phases. In the limit where $m'_1 = m'_2$ holds (i.e., both $m_1 = m_2$ and $\phi_1 = \phi_2$ hold [1]), one may transform the neutrino mass eigenstates in a simple way similar to Eq. (11),

$$\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}' \Longrightarrow \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}'' = \tilde{O}_{12} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}' . \tag{14}$$

This leads to $U'_{\mathrm{MNS}} = U_{\mathrm{MNS}}\tilde{O}^T_{12}$ in the new basis. Once parametrizations (A), (B), (J) and (L) in Table I are taken into account, we arrive at the corresponding patterns of U'_{MNS} as follows:

$$U'_{\mathrm{MNS}} = \begin{cases} O_{12} \otimes O_{23} \otimes P_1 & (\mathrm{A}) , \\ O_{12} \otimes O_{13} \otimes P_2 & (\mathrm{B}) , \\ O_{23} \otimes O_{13} \otimes P_2 & (\mathrm{J}) , \\ O_{13} \otimes O_{23} \otimes P_1 & (\mathrm{L}) , \end{cases} \tag{15}$$

in which $(U'_{\mathrm{MNS}})_{31} = 0$, $(U'_{\mathrm{MNS}})_{32} = 0$, $(U'_{\mathrm{MNS}})_{12} = 0$ and $(U'_{\mathrm{MNS}})_{21} = 0$, respectively. This result means that one of the four off-diagonal matrix elements in the first two columns of U_{MNS} or V_{MNS} (or equivalently, one of the three mixing angles of V_{MNS}) is removable in the $m'_1 = m'_2$ limit. Note that the complex phase in U'_{MNS}, which comes from P_1 or P_2, cannot be removed by redefining the mass eigenstates of charged leptons and neutrinos. Note also that the neutrino mass term in Eq. (13) consists of an irremovable Majorana phase in the $m'_1 = m'_2$ limit (i.e., the phase difference $\phi_3 - \phi_1 = \phi_3 - \phi_2$). Thus V_{MNS} would totally contain two mixing angles and two CP-violating phases in the conceptually interesting case where $m'_1 = m'_2$ held. Such a situation is quite different from the corresponding situation of quark flavor mixing.

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[1] Because P_ϕ is always associated with U_{MNS} for a given parametrization of V_{MNS}, the assumption of Majorana phase degeneracy (such as $\phi_1 = \phi_2$) is actually dependent on which explicit representation or phase convention has been adopted for U_{MNS}.

The above arguments can be repeated for $m_1' = m_3'$ and $m_2' = m_3'$ cases. We summarize the relevant results in Table IV. It is worth remarking that V_{MNS} may have one nontrivial mixing angle and one nontrivial CP-violating phase even in the $m_1' = m_2' = m_3'$ limit [15]. To see this point in a more obvious way, let us replace the transformation in Eq. (14) by

$$\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}' \Longrightarrow \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}'' = O_{ij}\tilde{O}_{12}\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}' , \tag{16}$$

where $O_{ij} = O_{23}$ for parametrizations (A) and (L) or $O_{ij} = O_{13}$ for parametrizations (B) and (J). Then we obtain $U_{\text{MNS}}' = U_{\text{MNS}}\tilde{O}_{12}^T O_{ij}^T$ in the new basis. Because O_{23} (or O_{13}) and P_1 (or P_2) are commutable, U_{MNS}' can be simplified to

$$U_{\text{MNS}}' = \begin{cases} O_{12} \otimes P_1 & \text{(A)} , \\ O_{12} \otimes P_2 & \text{(B)} , \\ O_{23} \otimes P_2 & \text{(J)} , \\ O_{13} \otimes P_1 & \text{(L)} . \end{cases} \tag{17}$$

It should be noted that three phases ϕ_i can all be removed from the neutrino mass term in the $m_1' = m_2' = m_3'$ limit (i.e., $\phi_1 = \phi_2 = \phi_3$). Hence we are left with one mixing angle and one CP-violating phase in this special case, as shown by Eq. (17).

Finally, we take a look at the case in which a pair of charged lepton masses are degenerate and a pair of complex (primed) neutrino masses are also degenerate. Let us assume $m_e = m_\mu$ and $m_1' = m_2'$ as an example. Then the orthogonal matrix on the left-hand side of U_{MNS}' in Eq. (15) can be rotated away by a proper redefinition of the mass eigenstates of charged leptons. Since O_{23} (or O_{13}) and P_1 (or P_2) are commutable, the latter can also be rotated away by rephasing the charged lepton fields. We are therefore left with the reduced MNS matrix $U_{\text{MNS}}'' = O_{23}$ for parametrizations (A) and (L) or $U_{\text{MNS}}'' = O_{13}$ for parametrizations (B) and (J). Taking account of the nontrivial Majorana phase in the neutrino mass term (i.e., the phase difference $\phi_3 - \phi_1 = \phi_3 - \phi_2$), we conclude that the MNS matrix would consist of one mixing angle and one CP-violating phase in the limit where both $m_e = m_\mu$ and $m_1' = m_2'$ held. The above argument can be repeated for any pair of charged leptons and any pair of neutrinos, and the relevant results are summarized in Table V.

Although there is little direct information about the absolute masses of three neutrinos, the observed phenomena of solar and atmospheric neutrino oscillations *do* forbit any pair of neutrino masses to be exactly degenerate. In addition, the strong mass hierarchy of three charged leptons has long been observed [3]. Thus the above discussions mainly serve for the conceptual clarification of lepton flavor mixing in the limit of lepton mass degeneracy.

IV. CONCLUDING REMARKS

We have done a systematic analysis of flavor mixing and CP violation in the limit where two quarks or leptons of the same charge are degenerate in mass. In particular, the impact of neutrino mass degeneracy and Majorana phase degeneracy on the lepton flavor mixing matrix has been discussed in some detail.

Although a limit of fermion mass degeneracy is primarily of conceptual interest, it might be able to serve as a useful starting point of view for building realistic models of flavor mixing. For instance, the degeneracy of three neutrino masses implies a possible S(3) symmetry in the effective Majorana neutrino mass matrix [16]: the observed neutrino mass-squared differences may be obtained by breaking that symmetry in a perturbative way, and a nearly bi-maximal lepton flavor mixing pattern is also achievable if the charged lepton mass matrix has an approximate (broken) $S(3)_L \times S(3)_R$ symmetry.

Understanding the generation of fermion masses and the origin of flavor mixing and CP violation has been a big challenge to particle physicists. Our analysis shows that the phenomenon of flavor mixing and that of CP violation would become simpler if three quarks or leptons of the same charge were partially or totally degenerate in mass. In this sense, possible correlation between fermion masses and flavor mixing is expected to exist in a theory more fundamental than the standard model. So is possible correlation between flavor mixing and CP violation. We hope that much more experimental progress in flavor physics could finally help us pin down the dynamics of fermion masses, flavor mixing and CP violation.

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TABLE I. Twelve different parametrizations of the CKM matrix V_{CKM} and the MNS matrix V_{MNS}. Here O_{ij} and $\tilde{O}_{ij}$ (for $ij = 12, 13, 23$) both describe a rotation in the (i, j) plane, but their corresponding rotation angles are in general different. The phase matrices P_i (for $i = 1, 2, 3$) and P_ϕ are defined as $P_1 = \mathrm{Diag}\{e^{i\delta}, 1, 1\}$, $P_2 = \mathrm{Diag}\{1, e^{i\delta}, 1\}$, $P_3 = \mathrm{Diag}\{1, 1, e^{i\delta}\}$ and $P_\phi = \mathrm{Diag}\{e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}\}$, respectively. Two of the three phases in P_ϕ or their combinations represent the two nontrivial Majorana-type CP-violating phases of V_{MNS}.

	V_{CKM}	V_{MNS}
Parametrization (A)	$O_{12} \otimes O_{23} \otimes P_1 \otimes \tilde{O}_{12}$	$O_{12} \otimes O_{23} \otimes P_1 \otimes \tilde{O}_{12} \otimes P_\phi$
Parametrization (B)	$O_{12} \otimes O_{13} \otimes P_2 \otimes \tilde{O}_{12}$	$O_{12} \otimes O_{13} \otimes P_2 \otimes \tilde{O}_{12} \otimes P_\phi$
Parametrization (C)	$O_{23} \otimes O_{12} \otimes P_3 \otimes \tilde{O}_{23}$	$O_{23} \otimes O_{12} \otimes P_3 \otimes \tilde{O}_{23} \otimes P_\phi$
Parametrization (D)	$O_{23} \otimes O_{13} \otimes P_2 \otimes \tilde{O}_{23}$	$O_{23} \otimes O_{13} \otimes P_2 \otimes \tilde{O}_{23} \otimes P_\phi$
Parametrization (E)	$O_{13} \otimes O_{12} \otimes P_3 \otimes \tilde{O}_{13}$	$O_{13} \otimes O_{12} \otimes P_3 \otimes \tilde{O}_{13} \otimes P_\phi$
Parametrization (F)	$O_{13} \otimes O_{23} \otimes P_1 \otimes \tilde{O}_{13}$	$O_{13} \otimes O_{23} \otimes P_1 \otimes \tilde{O}_{13} \otimes P_\phi$
Parametrization (G)	$O_{12} \otimes O_{23} \otimes P_1 \otimes O_{13}$	$O_{12} \otimes O_{23} \otimes P_1 \otimes O_{13} \otimes P_\phi$
Parametrization (H)	$O_{12} \otimes O_{13} \otimes P_2 \otimes O_{23}$	$O_{12} \otimes O_{13} \otimes P_2 \otimes O_{23} \otimes P_\phi$
Parametrization (I)	$O_{23} \otimes O_{12} \otimes P_3 \otimes O_{13}$	$O_{23} \otimes O_{12} \otimes P_3 \otimes O_{13} \otimes P_\phi$
Parametrization (J)	$O_{23} \otimes O_{13} \otimes P_2 \otimes O_{12}$	$O_{23} \otimes O_{13} \otimes P_2 \otimes O_{12} \otimes P_\phi$
Parametrization (K)	$O_{13} \otimes O_{12} \otimes P_3 \otimes O_{23}$	$O_{13} \otimes O_{12} \otimes P_3 \otimes O_{23} \otimes P_\phi$
Parametrization (L)	$O_{13} \otimes O_{23} \otimes P_1 \otimes O_{12}$	$O_{13} \otimes O_{23} \otimes P_1 \otimes O_{12} \otimes P_\phi$

TABLE II. The impact of quark mass degeneracy on the CKM matrix. The symbol "$\sqrt{}$" means that it is possible to make one off-diagonal matrix element of V_{CKM} vanishing in the limit where two quarks of the same charge are degenerate in mass.

	$m_u = m_c$	$m_u = m_t$	$m_c = m_t$	$m_d = m_s$	$m_d = m_b$	$m_s = m_b$
$V'_{\mathrm{CKM}} = O_{23} \otimes O_{13}$ with $(V'_{\mathrm{CKM}})_{12} = 0$	$\sqrt{}$	$\sqrt{}$		$\sqrt{}$		$\sqrt{}$
$V'_{\mathrm{CKM}} = O_{13} \otimes O_{23}$ with $(V'_{\mathrm{CKM}})_{21} = 0$	$\sqrt{}$		$\sqrt{}$	$\sqrt{}$	$\sqrt{}$	
$V'_{\mathrm{CKM}} = O_{23} \otimes O_{12}$ with $(V'_{\mathrm{CKM}})_{13} = 0$	$\sqrt{}$	$\sqrt{}$			$\sqrt{}$	$\sqrt{}$
$V'_{\mathrm{CKM}} = O_{12} \otimes O_{23}$ with $(V'_{\mathrm{CKM}})_{31} = 0$		$\sqrt{}$	$\sqrt{}$	$\sqrt{}$	$\sqrt{}$	
$V'_{\mathrm{CKM}} = O_{13} \otimes O_{12}$ with $(V'_{\mathrm{CKM}})_{23} = 0$	$\sqrt{}$		$\sqrt{}$		$\sqrt{}$	$\sqrt{}$
$V'_{\mathrm{CKM}} = O_{12} \otimes O_{13}$ with $(V'_{\mathrm{CKM}})_{32} = 0$		$\sqrt{}$	$\sqrt{}$	$\sqrt{}$		$\sqrt{}$

TABLE III. The impact of quark mass degeneracy on the CKM matrix. The symbol "$\sqrt{}$" means that it is possible to make two of the three mixing angles of V_{CKM} vanishing in the limit where a pair of up-type quarks and a pair of down-type quarks are respectively degenerate in mass.

	$V''_{\mathrm{CKM}} = O_{12}$	$V''_{\mathrm{CKM}} = O_{23}$	$V''_{\mathrm{CKM}} = O_{13}$
$m_u = m_c$ & $m_d = m_s$		$\sqrt{}$	$\sqrt{}$
$m_u = m_c$ & $m_d = m_b$		$\sqrt{}$	
$m_u = m_c$ & $m_s = m_b$			$\sqrt{}$
$m_u = m_t$ & $m_d = m_s$		$\sqrt{}$	
$m_u = m_t$ & $m_d = m_b$	$\sqrt{}$	$\sqrt{}$	
$m_u = m_t$ & $m_s = m_b$	$\sqrt{}$		
$m_c = m_t$ & $m_d = m_s$			$\sqrt{}$
$m_c = m_t$ & $m_d = m_b$	$\sqrt{}$		
$m_c = m_t$ & $m_s = m_b$	$\sqrt{}$		$\sqrt{}$

TABLE IV. The impact of lepton mass degeneracy on the MNS matrix. The symbol "$\sqrt{}$" means that it is possible to make one off-diagonal matrix element of V_{MNS} vanishing in the limit where two charged leptons are degenerate in mass or two neutrinos have the same mass and the same Majorana phase. Note that the complex neutrino masses $m'_a \equiv m_a e^{2i\phi_a}$ (for $a = 1, 2, 3$) contain the information about Majorana-type CP-violating phases.

	$m_e = m_\mu$	$m_e = m_\tau$	$m_\mu = m_\tau$	$m'_1 = m'_2$	$m'_1 = m'_3$	$m'_2 = m'_3$
$U'_{\mathrm{MNS}} = O_{23} \otimes O_{13}$ with $(U'_{\mathrm{MNS}})_{12} = 0$	$\sqrt{}$	$\sqrt{}$				
$U'_{\mathrm{MNS}} = O_{13} \otimes O_{23}$ with $(U'_{\mathrm{MNS}})_{21} = 0$	$\sqrt{}$		$\sqrt{}$			
$U'_{\mathrm{MNS}} = O_{23} \otimes O_{12}$ with $(U'_{\mathrm{MNS}})_{13} = 0$	$\sqrt{}$	$\sqrt{}$				
$U'_{\mathrm{MNS}} = O_{12} \otimes O_{23}$ with $(U'_{\mathrm{MNS}})_{31} = 0$		$\sqrt{}$	$\sqrt{}$			
$U'_{\mathrm{MNS}} = O_{13} \otimes O_{12}$ with $(U'_{\mathrm{MNS}})_{23} = 0$	$\sqrt{}$		$\sqrt{}$			
$U'_{\mathrm{MNS}} = O_{12} \otimes O_{13}$ with $(U'_{\mathrm{MNS}})_{32} = 0$		$\sqrt{}$	$\sqrt{}$			
$U'_{\mathrm{MNS}} = O_{23} \otimes O_{13} \otimes P_2$ with $(U'_{\mathrm{MNS}})_{12} = 0$				$\sqrt{}$		$\sqrt{}$
$U'_{\mathrm{MNS}} = O_{13} \otimes O_{23} \otimes P_1$ with $(U'_{\mathrm{MNS}})_{21} = 0$				$\sqrt{}$	$\sqrt{}$	
$U'_{\mathrm{MNS}} = O_{23} \otimes O_{12} \otimes P_3$ with $(U'_{\mathrm{MNS}})_{13} = 0$					$\sqrt{}$	$\sqrt{}$
$U'_{\mathrm{MNS}} = O_{12} \otimes O_{23} \otimes P_1$ with $(U'_{\mathrm{MNS}})_{31} = 0$				$\sqrt{}$	$\sqrt{}$	
$U'_{\mathrm{MNS}} = O_{13} \otimes O_{12} \otimes P_3$ with $(U'_{\mathrm{MNS}})_{23} = 0$					$\sqrt{}$	$\sqrt{}$
$U'_{\mathrm{MNS}} = O_{12} \otimes O_{13} \otimes P_2$ with $(U'_{\mathrm{MNS}})_{32} = 0$				$\sqrt{}$		$\sqrt{}$

TABLE V. The impact of quark mass degeneracy on the MNS matrix. The symbol "$\surd$" means that it is possible to make two of the three mixing angles of V_{MNS} vanishing in the limit where two charged leptons are degenerate in mass and two neutrinos have the same mass and the same Majorana phase. Note that the complex neutrino masses $m'_a \equiv m_a e^{2i\phi_a}$ (for $a = 1, 2, 3$) contain the information about Majorana-type CP-violating phases.

	$U''_{\mathrm{MNS}} = O_{12}$	$U''_{\mathrm{MNS}} = O_{23}$	$U''_{\mathrm{MNS}} = O_{13}$
$m_e = m_\mu$ & $m'_1 = m'_2$		$\surd$	$\surd$
$m_e = m_\mu$ & $m'_1 = m'_3$		$\surd$	
$m_e = m_\mu$ & $m'_2 = m'_3$			$\surd$
$m_e = m_\tau$ & $m'_1 = m'_2$		$\surd$	
$m_e = m_\tau$ & $m'_1 = m'_3$	$\surd$	$\surd$	
$m_e = m_\tau$ & $m'_2 = m'_3$	$\surd$		
$m_\mu = m_\tau$ & $m'_1 = m'_2$			$\surd$
$m_\mu = m_\tau$ & $m'_1 = m'_3$	$\surd$		
$m_\mu = m_\tau$ & $m'_2 = m'_3$	$\surd$		$\surd$